INVESTIGATIONS OF THE MIYATA SYNTHESIS TECHNIQUE

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THESIS

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ABSTRACT

These investigations generalize Miyata's synthesis of passive driving-point impedance functions by developing a step-by-step computational technique for augmenting a general driving-point impedance to insure that the real part of the augmented impedance is positive term by term. This goal has been accomplished and programmed under the provision that the real part of the original impedance is minimum reactive, i.e., has no zeros on the jω-axis. The latter requirement is not restrictive since zeros on the jω-axis can be removed a priori.

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I. INTRODUCTION

The Miyata[7,8] method of transformerless synthesis provides a method for separating a given impedance into additive components which individually satisfy certain special conditions that permit their realization in simple terms. This method of separation is based upon the even part of the given impedance and can be stated briefly as follows: Given a minimum reactive driving point impedance Z(s), find R(ω) the real part of Z(s). Then split R(ω) into a sum of even functions $R_p(\omega)$, (p=0,1,...,n) each satisfying the requirements of $\mathrm{Re}Z_p(j\omega)$. Find $Z_p(s)$ corresponding to each $R_p(\omega)$. The realization of Z(s) reduces to the realization of each of the $Z_p(s)$ and the series connection of the resulting one ports completes the synthesis.

An inherent difficulty in the above technique lies in the requirement that the $R_p(\omega)$ be positive for all ω. To overcome this difficulty Miyata suggested the use of surplus factors. An example will illustrate this method.

Let Z(s) be given as

$$Z(s) = \frac{6s^2+9s+9}{s^2+3s+4} \tag{1.1}$$

from which we obtain

$$R(\omega) = \frac{6\omega^4-6\omega^2+36}{B(\omega^2)} = R_2(\omega) + R_1(\omega) + R_0(\omega) \tag{1.2}$$

This so called "single-n split" method cannot be applied because $R_1(\omega)$ is negative. Therefore use a surplus factor (s+1) to obtain an augmented impedance, $Z_a(s)$.

$$Z_a(s) = \frac{s+1}{s+1}\, Z(s) = \frac{6s^3+15s^2+18s+9}{s^3+4\ s^2+7s+4} \tag{1.3}$$

for which the numerator of $\mathrm{Re}Z_a(j\omega)$ contains only positive terms.

$$R(\omega) = \frac{6\omega^6+30\omega^2+36}{B'(\omega^2)} \qquad (1.4)$$

Obviously the use of surplus factors complicates matters since higher-order factors are often required.

At this point most authors[1,8] considering the technique, including Miyata, generally abandon the use of surplus factors without discussing any general method for their selection and proceed to describe various groupings of the individual positive and negative $R_p(\omega)$ which remain positive for all ω. These grouping methods generally require factorization of the numerator with its attendant difficulties and no general method is available in the literature nor is it clear that such groupings are always possible.

The purpose of this thesis is to investigate in detail the use of surplus factors to develop augmented impedances that are positive term by term.

Chapter two is a general consideration of the Miyata method which describes the reasoning behind the technique and the mechanics of developing the electric circuit and component values from the original driving-point impedance when all terms in the numerator of $\mathrm{Re}Z(j\omega)$ are initially positive. The significant results of this section include:

The circuit realization is transformerless, an important advantage over the Bott-Duffin technique.

The complete circuit development consists of a simple unbalanced ladder network with series L's and shunt C's terminated in a resistance.

By term grouping without factorization, the number of circuit elements required is less than for the modified Bott-Duffin for all driving-point (dp) impedance functions of second order or greater. The saving in

number of elements becomes quite dramatic for higher-order polynomials so that even though the order of the original dp impedance must often be increased by the use of surplus factors, large savings are available.

In general, chapter two is included to provide sufficient incentive to undertake the task of developing a general method of selecting surplus factors of minimum order.

In chapter three, starting with a dp impedance that is the ratio of two general polynomials of the same order in s and a general augmenting polynomial (surplus factor), the real part of Z(s) is developed. Since the resultant coefficients of ω^{2n} must be non-negative, these coefficients are the required constraints that must be satisfied to insure that ReZ($j\omega$) is negative term by term. These coefficient constraints constitute a set of non-linear inequalities in the variables δ_i ($i=0,1,...,n$) which are the coefficients of the augmenting polynomial (i.e., $P(s)=\delta_o+\delta_i s+...+\delta_n s^n$). Also in this section, the non-linear inequalities developed are linearized using a set of congruent[9] transformations and solved by the simplex[2] method. The difficulties involved in the inverse transformation are circumvented by considering the form of the solved constraints which are the coefficients of the real part of the augmented dp impedance. The process is completed by demonstrating how a knowledge of the form of the solved constraints allows one to find the numerator of the real part by simple long division. Determination of the denominator is straightforward and the problem is solved.

A digital computer program for part of the routine described above is developed in chapter four, and a sample problem worked in section five.

In summary, a method has been developed which will invariable find an optimal augmenting polynomial such that the resultant augmented dp

impedance has a real part which is positive term by term. The method is optimal in the sense that as many coefficients as possible of the real part are zero, thereby reducing the required number of circuit elements to a minimum.

The value of this method lies in the fact that it will invariably produce a positive real part for the augmented impedance in a step-by-step fashion. In fact, the entire process could be programmed such that for any input minimum-reactive dp impedance the output would be the required circuit with accompanying element values.

The single major disadvantage is that the augmented dp impedance may have a large number of terms which would consequently require a large number of circuit elements to realize (the circumstances that determine this possibility are discussed in chapter two).

II. GENERAL CONSIDERATION OF THE MIYATA SYNTHESIS TECHNIQUE

The Miyata method for realization of a passive, minimum-reactive, driving-point impedance proceeds as follows:[1,7] For an impedance having no $j\omega$-axis zeros or poles written

$$Z(s) = \frac{m_1 + n_1}{m_2 + n_2} \tag{2.1}$$

where m denotes an even polynomial in s and n denotes an odd polynomial in s.

Its real part for $s=j\omega$ is

$$\operatorname{Re}Z(j\omega) = \frac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2} \tag{2.2}$$

or

$$\operatorname{Re}Z(j\omega) = \frac{A_o + A_1\omega^2 + \ldots + A_n\omega^{2n}}{B_o + B_1\omega^2 + \ldots + B_n\omega^{2n}} = \frac{A_o}{B(\omega^2)} + \frac{A_1\omega^2}{B(\omega^2)} + \cdot \cdot + \frac{A_n\omega^{2n}}{B(\omega^2)} \tag{2.3}$$

Each of the terms on the right-hand side, when the coefficients $A_o, A_1, \ldots, A_n$ are positive, may be shown to generate an impedance that can be realized without mutual inductive coupling. The series connections of such separate realizations yields the desired synthesis of $Z(s)$.

To see why this is so, consider the component impedances of the real part. The first term has n double-order zeros at s= infinity; the second has one double-order zero at s=0 and n-1 zeros at s= infinity; the third term has two double-order zeros at s=0 and n-2 zeros at s= infinity; finally, the last term has all its n double-order zeros at s=0. Therefore, all of these terms have their double-order zeros located exclusively at s=0 and s= infinity.

When the real part of an impedance is zero at some $j\omega$-axis point, it need not necessarily follow that the entire impedance is zero. However, the $j\omega$-axis points under consideration are restricted to s=0 and s= infinity and, since a minimum-reactive function must be purely real at these points, it follows that when the real part is zero the whole impedance must necessarily be zero.

Next we use the Darlington[1] theory to show that these impedances are completely developable in simple terms. The Darlington theory shows that in a cascade development of an impedance each component lossless network produces one set of zeros out of the total complement of zeros which the even part of that impedance contains. Therefore, the network generated by the first cycle provides one set of zeros and leaves a remainder whose even part contains the remaining even-part zeros. The next network in the chain provides another set and leaves a second remainder with the leftover even-part zeros - and so forth.

Applying this reasoning to the real part of the first term of the expansion of the real past of Z(s), we can predict that the corresponding impedances will have a zero at s= infinity and its reciprocal a pole at this point. Removal of this pole as a shunt capacitor is one Darlington cycle. The admittance remainder after this pole at s= infinity is removed has an even part that still has n-1 double-order zeros at s= infinity since so far only one has been used up. This admittance remainder must have a zero at s= infinity and its reciprocal must have a pole there. Removal of this pole as a series inductor constitutes the second cycle. The remainder now has an even part with n-2 double-order zeros at s= infinity and is zero at s= infinity. Its reciprocal must have a pole at this point and its removal as a shunt capacitance completes

the third cycle. This process continues yielding series inductive and shunt capacitive branches of total number n, the number of double-order zeros possessed by the original real part. When the last of these zeros has been accounted for, the remainder is reduced to a constant because its real part has no more zeros. The complete development consists of a simple unbalanced ladder network with series inductances and shunt capacitances terminated in a resistance. Subsequent terms are similarly developed.

Using this single-n split technique, the total number of reactive elements is n^2, where n equals the highest degree of either the numerator or denominator polynomial in s. Comparison with the number of elements required for a Bott-Duffin synthesis is displayed in Table I along with the reductions due to the 1/2-n split technique which is subsequently discussed. The geometric character of the Bott-Duffin method compared with the arithmetic character of the Miyata method is strongly apparent in this table. Even though special devices are required to salvage the Miyata method when all the terms of the real part numerator are not initially positive, there appears to be considerable advantage to exploring the Miyata procedure where more elaborate impedance functions are involved. Before one gets too enthusiastic about the Miyata method, it would be well to consider the requirement that all the coefficients in the polynomial formed by the numerator of the real part of the driving-point impedance must be positive.

A sufficient condition that the coefficients be positive can be obtained by recalling that a Hurwitz polynomial has positive coefficients. Thus if

$$A(\omega^2) = A_0 + \ldots + A_n\omega^{2n} \qquad (2.4)$$

has no zeros in the right-half ω^2 plane, then $A_0,\ldots,A_n$ are positive. Or if

$$A(-s^2) = m_1m_2 - n_1n_2 \tag{2.5}$$

has no zeros in the left-half s^2 plane, then the $A_0,\ldots,A_n$ are positive. Since the left-half s^2 plane maps into a region of the s plane containing all points not more than plus or minus 45^o from the $j\omega$-axis, we can say the coefficients in eq. 2.4 are positive so long as there are no even-part zeros of Z(s) closer than 45^o to the $j\omega$-axis.

Although the coefficients in 2.4 may still be positive when some even-part zeros lie closer to the $j\omega$-axis than this sufficiency condition permits, the chances of this desired condition being fulfilled when even-part zeros are relatively close to the $j\omega$-axis become remote and vanish completely when a single pair of even-part zeros lie upon the $j\omega$-axis.

In a manner of speaking, we can say that a polynomial that has some zeros above the 45^o line in the s plane may still yield all positive coefficients if there are a sufficient number of zeros below the 45^o lines to produce a compensating effect. Therefore, in a situation in which the even part of Z(s) has some negative coefficients, we can augment the real part with appropriately chosen factors to bring about the desired compensation.

The method of compensation fails if the even part of Z(s) has any $j\omega$-axis zeros, since the number of necessary compensating factors tends toward infinity as one or more pairs of even-part zeros approach the $j\omega$-axis. With such a compensating scheme the number of terms in the real part and hence the number of necessary elements increases rapidly.

One method to reduce the number of elements is considered now, the 1/2-n split.[1,8] Miyata points out that the detailed procedure for carrying out the basic method of decomposition may be varied by separating the real-part numerator polynomial into groups of terms rather than separate terms.

Consider

$$\mathrm{Re}Z(j\omega) = \frac{(A_o + A_1\omega^2 + \ldots + A_{n-k}\omega^{2(n-k)})}{B(\omega^2)} + \frac{(A_{n-k+1}\omega^{2(n-k+1)} + \ldots + A_n\omega^{2n})}{B(\omega^2)} \tag{2.6}$$

where $k=n/2$ (n even) $k=(n+1)/2$ (n odd).

S. N. Hunt[6], who refers to this decomposition as the 1/2-n split, finds that it yields a smaller total number of elements than any other division of the numerator polynomial into parts. It requires fewer elements than the Bott-Duffin method for any degree n equal to or greater than 2. The impedance generated by the first term is developable into a simple unbalance ladder with series L's and shunt C's (total number k) terminated in a remainder whose even-part zeros are the same finite non-zero ones contained in the even part of this impedance. The second term is developable into an unbalanced ladder with series C's and shunt L's (total number n-k+1) and a terminal impedance whose even-part zeros are those that are contained in the even part of the second impedance. To the terminal impedances which appear as remainder functions we can apply a similar treatment and continue until the ultimate remainder functions are resistances.

The process of augmentation by surplus factors is also necessary in order that Miyata's elegant method of determining an impedance from its real part may be used. The method proceeds as follows:[7,8]

$$Z(s) = \frac{m_1 + n_1}{m_2 + n_2} \tag{2.7}$$

and

$$\mathrm{Ev}Z(s) = \frac{m_1m_2 - n_1n_2}{m_2^{\,2} - n_2^{\,2}} = \frac{N(s)}{D(s)} \; . \tag{2.8}$$

Next consider an auxiliary function

$$Z'(s) = \frac{N(m_1' + n_1')}{m_2 + n_2} \; . \tag{2.9}$$

Its even part is

$$\mathrm{Ev}Z'(s) = \frac{N(m_1'm_2 - n_1'n_2)}{m_2^{\,2} - n_2^{\,2}} \; . \tag{2.10}$$

Set the two even parts to be equal;

$$\mathrm{Ev}Z'(s) = \mathrm{Ev}Z(s) \tag{2.11}$$

which can be done provided

$$m_1'm_2 - n_1'n_2 = 1. \tag{2.12}$$

where the degree of $m_1'+n_1'$ is that of m_1+n_1. Polynomials m_1' and n_1' can always be found to satisfy this requirements. $Z'(s)$ however will have a numerator of degree two or more higher than its denominator and is therefore not positive real (p.r.). To find the p.r. function $Z(s)$, carry out a long division on $Z'(s)$ until the degree of the numerator does not exceed the degree of the denominator. That is,

$$Z'(s) = \frac{N(m_1' + n_1')}{m_2 + n_2} = q(s) + \frac{m_r + n_r}{m_2 + n_2} \tag{2.13}$$

where $q(s)$ is the quotient polynomial and m_r+n_r is the remainder.

Let $Z_r(s)$ be

$$Z_r(s) = \frac{m_r + n_r}{m_2 + n_2} \tag{2.14}$$

so that

$$Z(s) = Z_r(s) + q(s) \tag{2.15}$$

Recalling that $EvZ'(s) = EvZ(s)$,

$$EvZ(s) = EvZ_r(s) + Evq(s). \tag{2.16}$$

If the polynomial $q(s)$ contains an even power of s then the right hand side of eq. 2.16 tends to infinity as $s=j\omega$ tends to infinity. On the other hand, $EvZ(s)_{s=j\omega} = R(\omega)$ is bounded at infinity. Therefore $q(s)$ must be an odd polynomial and as a result

$$EvZ_r(s) = EvZ(s). \tag{2.17}$$

Since the numerator of $Z_r(s)$ does not exceed the denominator $Z_r(s)$ has no pole at infinity or at any other finite frequency on the $j\omega$-axis since its denominator is that of $Z(s)$.

Briefly $Z_r(s)$ has no poles in the right-half plane or on the $j\omega$-axis, and its even part for $s=j\omega$ is always non-negative. Therefore $Z_r(s)$ is positive real. To illustrate:

$$\text{Given } ReZ_o(j\omega) = \frac{1}{1+\omega^6} \tag{2.17a}$$

$$Z_o(s) = \frac{2/3s^2 + 4/3s + 1}{s^3 + 2s^2 + 2s + 1} \tag{2.17b}$$

from any method whatsoever. Suppose we want an impedance with the real part $ReZ_1(j\omega) = \omega^2/1+\omega^6$ which is 2.17a multiplied by ω^2 and implies $m=-s^2$.

Then $F(s)$ equals eq. 2.17b multiplied by $-s^2$ and the long division takes the form

$$\begin{array}{r|l} & -2/3s \\ \hline s^3 + 2s^2 + s + 1 & -2/3s^4 - 4/3s^3 - s^2 \\ & -2/3s^4 - 4/3s^3 - 4/3s^2 \\ \hline & \qquad\qquad 1/3s^2 + 2/3s \end{array}$$

and

$$Z_1(s) = \frac{1/3s^2 + 2/3s}{s^3 + 2s^2 + 2s + 1}$$

Using this procedure any $Z_n(s) = \omega^{2n}/(1 + \omega^6)$ n=1,2,... can be found from $Z_o(s)$. If we want an impedance with the real part

$$\mathrm{Re}Z(j\omega) = A_o + A_1\omega^2 + A_2\omega^4 + A_3\omega^6$$

Z(s) may be generated as

$$A_oZ_o(s) + A_1Z_1(s) + A_2Z_2(s) + A_3Z_3(s)$$

where continuing as above

$$Z_2(s) = \frac{2/3s^2 + 1/3s}{s^3 + 2s^2 + 2s + 1}$$

$$Z_3(s) = \frac{s^3 + 4/3s^2 + 2/3s}{s^3 + 2s^2 + 2s + 1}.$$

$Z_2(s)$ is found by multiplying $Z_1(s)$ by $-s^2$ and $Z_3(s)$ found by multiplying $Z_2(s)$ by $-s^2$ and performing the indicated long division.

III. GENERAL STEP-BY-STEP PROCEDURE FOR THE SELECTION OF SURPLUS FACTORS

The topic of this chapter is the step-by-step procedure for developing the real part of an augmented impedance that is positive term by term from a general n^{th}-order, passive, minimum-reactive, driving-point impedance augmented by a general m^{th}-order augmenting polynomial.

A. CONSTRAINT DEVELOPMENT

This section is devoted to the development of the required constraints to insure a positive real part for the dp impedance and begins with a general impedance function and augmenting polynomial as follows:

Consider first the general driving point impedance Z(s)

$$Z(s) = \frac{\sum\limits_{q=0}^{N} \alpha_q s^q}{\sum\limits_{q=0}^{N} \beta_q s^q} = \frac{N(s)}{D(s)} \tag{3.1}$$

The general augmenting polynomial P(s)

$$P(s) = \sum_{r=0}^{T} \delta_r s^r \tag{3.2}$$

The product of Z(s) and P(s)/P(s) is the augmented driving point impedance $Z_a(s)$;

$$Z_a(s) = \frac{\sum\limits_{q=0}^{N} (\alpha_q s^q \sum\limits_{r=0}^{T} \delta_r s^r)}{\sum\limits_{q=0}^{N} (\beta_q s^q \sum\limits_{r=0}^{T} \delta_r s^r)} \tag{3.3}$$

$$Z_a(s) = \frac{\sum\limits_{\substack{q=0\\r=0}}^{\substack{N\\T}} \alpha_q \delta_r s^{q+r}}{\sum\limits_{\substack{q=0\\r=0}}^{\substack{N\\T}} \beta_q \delta_r s^{q+r}} = \frac{m_1' + n_1'}{m_2' + n_2'} \tag{3.4}$$

The even part of the augmented driving point impedance is

$$\mathrm{Ev}Z_a(s) = \frac{m_1'm_2' - n_1'n_2'}{m_2'^2 - n_2'^2} = \frac{N'(s)}{D'(s)} \tag{3.5}$$

where

$$m_1' = \sum_{\substack{u=0\\r=0}}^{\substack{(N+T)/2\\T}} \alpha_{2u-r}\delta_r s^{2u} \qquad m_2' = \sum_{\substack{u=0\\r=0}}^{\substack{(N+T)/2\\T}} \beta_{2u-r}\delta_r s^{2u}$$

$$n_1' = \sum_{\substack{u=0\\r=0}}^{\substack{(N+T-1)/2\\T}} \alpha_{2u+1-r}\delta_r s^{2u+1} \qquad n_2' = \sum_{\substack{u=0\\r=0}}^{\substack{(N+T-1)/2\\T}} \beta_{2u+1-r}\delta_r s^{2u+1} \tag{3.6}$$

and noninteger values of $(N+T)/2$ and $(N+T-1)/2$ are rounded to the next lowest integer.

Expanding the even part of $Z_a(s)$

$$\mathrm{Ev}Z_a(s) = \frac{\sum_{\substack{u=0\\r=0}}^{\substack{(N+T)/2\\T}} \alpha_{2u-r}\delta_r s^{2u} \sum_{\substack{u=0\\r=0}}^{\substack{(N+T)/2\\T}} \beta_{2u-r}\delta_r s^{2u} - \sum_{\substack{u=0\\r=0}}^{\substack{(N+T-1)/2\\T}} \alpha_{2u+1-r}\delta_r s^{2u+1} \sum_{\substack{u=0\\r=0}}^{\substack{(NT-1)/2\\T}} \beta_{2u+1-r}\delta_r s^{2u+1}}{D'(s)} \tag{3.7}$$

Combining products,

$$\mathrm{Ev}Z_a(s) = \frac{\sum_{\substack{u,v=0\\q,r=0}}^{\substack{(N+T)/2\\T}} s^{2(u+v)}\alpha_{2v-r}\beta_{2u-q}\delta_r\delta_q - \sum_{\substack{u,v=0\\q,r=0}}^{\substack{(N+T-1)/2\\T}} s^{2(u+v+1)}\alpha_{2v+1-r}\beta_{2u+1-q}\delta_r\delta_q}{D'(s)} \tag{3.8}$$

Combining the difference,

$$\mathrm{Ev}Z_a(s) = \sum_{\substack{u,v=0\\r,q=0}}^{\substack{(N+T)/2\\T}} s^{2(u+v)} \frac{\delta_r\delta_q(\alpha_{2v-r}\beta_{2u-q} - s^2\alpha_{2v+1-r}\beta_{2u+1-q})}{D'(s)} \tag{3.9}$$

Since $\mathrm{Re}Z_a(j\omega) = \mathrm{Ev}Z_a(s)_{s=j\omega}$ and the Miyata method requires that the real part be positive term by term, constraints must be developed to insure this requirement. First, consider the denominator D'(s) above

$$D'(s) = m_2'^2 - n_2'^2 = D(j\omega)D(-j\omega) = |D(j\omega)|^2 \qquad (3.10)$$

which is never negative and is therefore of no concern.

Thus the constraint is reduced to

$$N'(\omega^2) = m_1'm_2' - n_1'n_2' \Big|_{s=j\omega} \geqq 0 \text{ for } 0 \leqq \omega \leqq \infty \qquad (3.11)$$

The coefficient of each term in eq. 3.9 must be non-negative which requires that

$$\sum_{\substack{m=0\\u=0\\r,q=0}}^{\substack{(N+T-1)/2\\(N+T)/2\\T}} s^{4m+2}\delta_r\delta_q(\alpha_{4m+2-2u-r}\beta_{2u-q} - \alpha_{4m+1-2u-r}\beta_{2u+1-q}) \leqq 0 \qquad (3.12)$$

and

$$\sum_{\substack{n=0\\u=0\\r,q=0}}^{\substack{(N+T-2)/2\\(N+T)/2\\T}} s^{4m+4}\delta_r\delta_q(\alpha_{4m+4-2u-r}\beta_{2u-q} - \alpha_{4m+3-2u-r}\beta_{2u+1-q}) \geqq 0 \qquad (3.13)$$

In equation 3.9 when the exponent $2(u+v) = 4m+2$ $(m=0,1,\ldots,(N+T-1)/2)$, the summation constraint becomes eq. 3.12; and when the exponent $2(u+v) = 4m+4$ $(m=0,1,\ldots,(N+T-2)/2)$, the summation constraint becomes eq. 3.13 since the summation must be non-negative when evaluated at $s=j\omega$.

Collecting terms and simplifying, eqs. 3.12 and 3.13 become

$$\sum_{u,r,q,m,=0} \delta_r\delta_q(\alpha_{\gamma-3}\beta_{\epsilon+1} - \alpha_{\gamma-2}\beta_\epsilon) \geqq 0 \qquad (3.14)$$

$$\sum_{u,r,q,m=0} \delta_r\delta_q(\alpha_\gamma\beta_\epsilon - \alpha_{\gamma-1}\beta_{\epsilon+1}) \geqq 0 \qquad (3.15)$$

where

$u=0,1,\ldots,(N+T-1)/2$	$u=0,1,\ldots,N+T/2$
$r=0,1,\ldots,T$	$r=0,1,\ldots,T$
$q=0,1,\ldots,T$	$q=0,1,\ldots,T$
$m=0,1,\ldots,N+T-1/2$	$m=0,1,\ldots,(N+T-2)/2$
(3.14)	(3.15)

and

$$\gamma = 4m+4-2u-4 \qquad \epsilon = 2u-q$$

N = Order of original impedance. T = order of augmenting polynomial.

The final value of the indices is chosen so that if the value is non-integer the rule is to round down to the next lower integer.

In the summations above, given values for α and β, δ's must be chosen so that these inequalities are satisfied.

Expanding the generalized constraints for a first-order augmenting polynomial (i.e., $T = 1$; $r,q = 0,1$)

$$\begin{aligned} \underline{A}\delta_0^{\ 2} + \underline{B}\delta_0\delta_1 + \underline{C}\delta_1^{\ 2} &\geqq 0 \\ \underline{D}\delta_0^{\ 2} + \underline{E}\delta_0\delta_1 + \underline{F}\delta_1^{\ 2} &\geqq 0 \end{aligned} \tag{3.16}$$

where

$$\begin{aligned} \underline{A} &= \alpha_{4m+1-2u}\beta_{2u+1} - \alpha_{4m+2-2u}\beta_{2u} \\ \underline{B} &= \alpha_{4m-2u}\beta_{2u+1} - \alpha_{4m+2-2u}\beta_{2u-1} \\ \underline{C} &= \alpha_{4m-2u}\beta_{2u} - \alpha_{4m+1-2u}\beta_{2u-1} \\ \underline{D} &= \alpha_{4m+4-2u}\beta_{2u} - \alpha_{4m+3-2u}\beta_{2u+1} \\ \underline{E} &= \alpha_{4m+4-2u}\beta_{2u-1} - \alpha_{4m+2-2u}\beta_{2u+1} \\ \underline{F} &= \alpha_{4m+3-2u}\beta_{2u-1} - \alpha_{4m+2-2u}\beta_{2u} \end{aligned} \tag{3.17}$$

$$\underline{A} = \begin{bmatrix} \alpha_1\beta_1 - \alpha_2\beta_0 - \alpha_0\beta_2 \\ \alpha_3\beta_3 + \alpha_1\beta_5 + \alpha_3\beta_1 - \alpha_2\beta_4 - \alpha_4\beta_2 - \alpha_0\beta_6 - \alpha_6\beta_0 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{3.18}$$

$$\underline{B} = \underline{0} \tag{3.19}$$

$$\underline{C} = \begin{bmatrix} \alpha_0\beta_0 \\ \alpha_4\beta_0 + \alpha_0\beta_4 + \alpha_2\beta_2 - \alpha_3\beta_1 - \alpha_1\beta_3 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{3.20}$$

$$\underline{D} = \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{3.20a}$$

$$\underline{E} = \underline{0} \tag{3.20b}$$

$$\underline{F} = \begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{3.20c}$$

and so for a first-order augmenting polynomial the constraints are

$$\begin{aligned} a_k\delta_0^{\,2} + c_k\delta_1^{\,2} &\geqq 0 \\ c_{k+1}\delta_0^{\,2} + a_k\delta_1^{\,2} &\geqq 0 \end{aligned} \tag{3.21}$$

where $k = 0,1,\ldots,(N-1)/2$ for the a's and $k = 0.1,\ldots,N/2$ for the c's; N is the order of the original driving point impedance.

Performing the expansions above for higher-order augmenting polynomials provides the results displayed in Table II.

The equations of Table II can be put into matrix format. Equation 3.21 becomes

$$\begin{bmatrix} \delta_0 & \delta_1 \end{bmatrix} \begin{bmatrix} a_k & 0 \\ 0 & c_k \end{bmatrix} \begin{bmatrix} \delta_0 \\ \delta_1 \end{bmatrix} = 0$$

$$\begin{bmatrix} \delta_0 & \delta_1 \end{bmatrix} \begin{bmatrix} c_{k+1} & 0 \\ 0 & a_k \end{bmatrix} \begin{bmatrix} \delta_0 \\ \delta_1 \end{bmatrix} = 0 \tag{3.22}$$

In general it can be shown that

$$\begin{aligned} \underline{\delta}^t \, \underline{Q}_j^k \, \underline{\delta} \geq 0 \\ \underline{\delta}^t \, \underline{R}_j^k \, \underline{\delta} \geq 0 \end{aligned} \tag{3.22a}$$

where superscript t indicates a transpose, $k = 0,1,\ldots,N/2$ and $j = T+1$ for the first-order example (T is the order of the augmenting polynomial).

$$\begin{bmatrix} a_k & 0 & -c_k & 0 & a_{k-1} & 0 \\ 0 & c_k & 0 & -a_k & 0 & c_{k-1} \\ -c_k & 0 & a_{k-1} & 0 & -c_{k-1} & 0 \\ 0 & -a_k & 0 & c_{k-1} & 0 & -a_{k-2} \\ a_{k-1} & 0 & -c_{k-1} & 0 & a_{k-2} & 0 \\ 0 & c_{k-1} & 0 & -a_{k-2} & 0 & c_{k-2} \end{bmatrix} = \underline{Q}_j^k \tag{3.23}$$

$$\begin{bmatrix} c_{k+1} & 0 & -a_k & 0 & c_k & 0 \\ 0 & a_k & 0 & -c_k & 0 & a_{k-1} \\ -a_k & 0 & c_k & 0 & -a_{k-1} & 0 \\ 0 & -c_k & 0 & a_{k-1} & 0 & -c_{k-1} \\ c_k & 0 & -a_{k-1} & 0 & c_{k-1} & 0 \\ 0 & a_{k-1} & 0 & -c_{k-1} & 0 & a_{k-2} \end{bmatrix} = \underline{R}_j^k \qquad (3.23)$$

Note that $\underline{Q}_j^k$ and $\underline{R}_j^k$ are dimensioned (T+1) x (T+1).

To illustrate the process so far developed consider the dp impedance used for the example in chapter V.

$$Z(s) = \frac{s^2 + 1/2s + 1}{s^2 + s + 1}$$

which requires a third-order augmenting polynomial.

$$T = 3 \qquad\qquad N = 2$$

$$\alpha_0 = 1 \qquad \alpha_1 = 1/2 \qquad \alpha_2 = 1$$

$$\beta_0 = 1 \qquad \beta_1 = 1 \qquad \beta_2 = 1$$

$$j = T + 1 = 4$$

$$k = 0 \text{ for } a\text{'s}$$

$$k = 0,1 \text{ for } c\text{'s}$$

$$a_0 = \alpha_1\beta_1 - \alpha_2\beta_0 - \alpha_0\beta_2 = -3/2$$

$$a_1 = 0$$

$$c_0 = \alpha_0\beta_0 = 1$$

$$c_1 = \alpha_2\beta_2 = 1$$

$$\underline{Q}_4^0 = \begin{bmatrix} -3/2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3/2 \\ -1 & 0 & 0 & 0 \\ 0 & 3/2 & 0 & 0 \end{bmatrix} \qquad \underline{Q}_4^1 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{R}_4^0 = \begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & -3/2 & 0 & -1 \\ 3/2 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \qquad \underline{R}_4^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -3/2 \end{bmatrix}$$

It remains to solve for the δ coefficients according to eq. (3.22) where

$$\delta = \begin{bmatrix} \delta_0 \\ \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

B. LINEARIZATION AND SOLUTION OF CONSTRAINTS[3,11,12]

In this section the non-linear inequalities developed in the preceding section are linearized using a set of congruent transformations and solved. Inspecting matrices $\underline{Q}$ and $\underline{R}$, the pattern of their formation becomes apparent. Note that they are symmetric for all orders. Evaluating $\underline{Q}_j^k$ and $\underline{R}_j^k$ for fixed j=T+1 over the indices k=0,1,...,N/2 produces a set of constraint inequalities as follow

$$\underline{\delta}^t \, \underline{Q}_j^k \, \underline{\delta} \geqq 0$$
$$\underline{\delta}^t \, \underline{R}_j^k \, \underline{\delta} \geqq 0 \qquad (3.24)$$

or

$$\begin{aligned} &\underline{\delta}^t \, \underline{Q}_j^0 \, \underline{\delta} \geqq 0 \\ &\underline{\delta}^t \, \underline{R}_j^0 \, \underline{\delta} \geqq 0 \\ &\quad \vdots \\ &\underline{\delta}^t \, \underline{R}_j^{N/2} \, \underline{\delta} \geqq 0 \end{aligned} \qquad (3.25)$$

Inequalities 3.25 represent N conditions involving T+1 unknowns in δ.(1)

Where

$$\underline{\delta} = \begin{bmatrix} \delta_0 \\ \delta_1 \\ \cdot \\ \cdot \\ \cdot \\ \delta_{j-1} \end{bmatrix}$$

As a first step toward the simultaneous solution of the set of inequalities above, consider the transforms for eq. 3.24.

$$\underline{\delta} = \underline{M}_j^k \, \underline{x}$$
$$\underline{\delta} = \underline{N}_j^k \, \underline{x} \qquad (3.26a)$$

where $\underline{M}_j^k$ ($\underline{N}_j^k$) is the normalized modal matrix of $\underline{Q}_j^k$ ($\underline{R}_j^k$), i.e., the matrix of normalized eigenvectors of $\underline{Q}_j^k$ ($\underline{R}_j^k$). These orthogonal

---

(1) Since one of the δ_j can be chosen arbitrarily, there are actually T unknowns in δ.

transforms convert the quadratic forms $\underline{\delta}^t \underline{Q}_j^k \underline{\delta}$ ($\underline{\delta}\ \underline{R}_j^k\ \underline{\delta}$) into a linear combination of the squares of the coordinates of the vector $\underline{x}$ with no cross products. Alternately the transform can be viewed as reducing the matrices $\underline{Q}_j^k$ ($\underline{R}_j^k$) to diagonal form;

i.e.,

$$\underline{\delta}^t \underline{Q}_j^k \underline{\delta} \tag{3.27}$$

under the transform

$$\delta = \underline{M}_j^k \underline{x} \tag{3.27a}$$

becomes

$$\underline{x}^t (\underline{M}_j^k) \underline{Q}_j^k \underline{M}_j^k \underline{x} = \underline{x}^t (\underline{M}_j^k)^{-1} \underline{Q}_j^k \underline{M}_j^k \underline{x} = 1_0 x_0^{\ 2} + 1_1 x_1^{\ 2} + \ldots + 1_{j-1} x_{j-1}^{\ 2} \tag{3.28}$$

since a property of an orthogonal matrix is $(M_j^k)^t = (M_j^k)^{-1}$. The 1's in eq. 3.28 are the eigenvalues of $\underline{Q}_j^k$, and $\underline{M}_j^k$ is the orthogonal matrix of normalized eigenvectors which are always available.

Using the transforms above in eq. 3.25 yields

$$\begin{aligned} \underline{x}^t (\underline{M}_j^k)^t \underline{Q}_j^k \underline{M}_j^k \underline{x} &\geqq 0 \\ \underline{x}^t (\underline{N}_j^k)^t \underline{R}_j^k \underline{N}_j^k \underline{x} &\geqq 0 \end{aligned} \tag{3.29}$$

Substituting

$$\begin{aligned} {}_q\underline{D}_j^k &= (\underline{M}_j^k)^t \underline{Q}_j^k \underline{M}_j^k \\ {}_r\underline{D}_j^k &= (\underline{N}_j^k) \underline{R}_j^k \underline{N}_j^k \end{aligned} \tag{3.29a}$$

where the D's are diagonal matrices, 3.29 becomes

$$\begin{aligned} \underline{x}^t\ {}_q\underline{D}_j^k\ \underline{x} &\geqq 0 \\ \underline{x}^t\ {}_r\underline{D}_j^k\ \underline{x} &\geqq 0 \end{aligned} \tag{3.30}$$

Writing the quadratic forms as the product of a row and column vector eq. 3.30 becomes

$$\begin{aligned} {}_{q}\underline{D}_{j}^{k} \text{ (row) } \underline{z} &\geqq 0 \\ {}_{r}\underline{D}_{j}^{k} \text{ (row) } \underline{z} &\geqq 0 \end{aligned} \tag{3.31}$$

where

$$z = \begin{bmatrix} x_0^2 \\ x_1^2 \\ \cdot \\ \cdot \\ \cdot \\ x_{j-1}^2 \end{bmatrix} \tag{3.31a}$$

and ${}_{q}\underline{D}_{j}^{k}$ (row) (${}_{r}\underline{D}_{j}^{k}$ (row)) indicate ${}_{q}\underline{D}_{j}^{k}$ (${}_{r}\underline{D}_{j}^{k}$) written as a row. There are N constraints on T+1 variables in $\underline{z}$.

The solution to this linearized set of inequalities can be quite elegantly found using Danzig's simplex method[2] modified to permit optimization according to the criteria that as many of the individual inequalities as possible set to zero. The simplex method initially determines if the set of inequalities is consistent. For the purpose of this thesis it is recommended that one start with T = 1. If the inequalities are inconsistent simply increase the order of the augmenting polynomial by one and proceed until a feasible set is generated. According to Guellemin[1], if the even part of Z(s) has no $j\omega$-axis zeros a finite feasible set can always be found.

A rigorous proof of this assertion is not available in the literature. Next the inverse transformation of eq. 3.26a must be accomplished in such a way that the resultant δ's are equal (i.e., $\delta_0=\delta_0$, $\delta_1=\delta_1$, etc.).

It should be noted that since $\underline{M}_j^k$ and $\underline{N}_j^k$ are different, different δ's can be expected for the same x's. In Eq. 3.27 postmultiplying the $\underline{M}_j^k$ ($\underline{N}_j^k$) by ${}_m\underline{D}^k$ (${}_n\underline{D}^k$), which are arbitrary diagonal matrices, doesn't alter the congruency of the transform (i.e., the quadratic form in $\underline{\delta}$ is still transformed to a linear combination of the squares of the co-ordinates of the vector $\underline{x}$). Since $\underline{M}_j^k$ ($\underline{N}_j^k$) is the normalized modal matrix, postmultiplying by an arbitrary diagonal matrix is equivalent to scaling the modal matrix.

Therefore, inverse transforming eq. 3.26a in such a way that the δ's are equal is equivalent to requiring that diagonal matrices ${}_m\underline{W}^k$ and ${}_n\underline{W}^k$ be found such that

$$\underline{M}_j^k \, {}_m\underline{W}^k \, \underline{x} = \underline{N}_j^k \, {}_n\underline{W}^k \, \underline{x} \tag{3.32}$$

where eq. 3.32 is developed by setting the two equations 3.26a, postmultiplied by the required diagonal matrices, equal to each other.

For the purposes of this thesis choose ${}_m\underline{W}^k = 1^k \, \underline{I}$ and ${}_n\underline{W}^k = p^k \underline{I}$, where 1^k and p^k are scalars and $\underline{I}$ is the identity matrix. With this substitution, eq. 3.32 becomes

$$1^k \, \underline{M}_j^k \, \underline{x} = p^k \, \underline{N}_j^k \, \underline{x} \tag{3.33}$$

or

$$\underline{M}_j^k \, \underline{x} = q^{k,k} \, \underline{N}_j^k \, \underline{x} \tag{3.33a}$$

where

$$q^{k,k} = 1^k \, p^k$$

Rewriting 3.33a by premultiplying both sides by $(\underline{N}_j^k)^t$ yields:

$$(\underline{N}_j^k)^t \, \underline{M}_j^k \, \underline{x} = q^{k,k} \, \underline{x} \tag{3.33b}$$

Expanding eq. 3.33b over the index k,(j = T+1) yields

$$
\begin{array}{cccc}
(\underline{N}_j^0)^t \, \underline{M}_j^0 \, \underline{x} = q^{0,0} \, \underline{x} & (\underline{N}_j^0)^t \, \underline{M}_j^1 \, \underline{x} = q^{0,1} \, \underline{x} & & (\underline{N}_j^0)^t \, \underline{M}_j^{N/2} \underline{x} = q^{0,N/2} \underline{x} \\
(\underline{N}_j^1)^t \, \underline{M}_j^0 \, \underline{x} = q^{1,0} \, \underline{x} & (\underline{N}_j^1)^t \, \underline{M}_j^1 \, \underline{x} = q^{1,1} \, \underline{x} & \cdots & (\underline{N}_j^1)^t \, \underline{M}_j^{N/2} \underline{x} = q^{1,N/2} \underline{x} \\
\vdots & \vdots & & \vdots \\
(\underline{N}_j^{N/2})^t \underline{M}_j^0 \, \underline{x} = q^{N/2,0} \underline{x} & (\underline{N}_j^{N/2})^t \underline{M}_j^1 \underline{x} = q^{N/2,1} \underline{x} & & (\underline{N}^{N/2})^t \underline{M}_j^{N/2} \underline{x} = q^{N/2,N/2} \underline{x}
\end{array}
\qquad (3.34)
$$

Equations 3.34 are the familiar eigenvalue problem in which solution constants $q^{k,k}$ (the eigenvalues) can always be found for $(\underline{N}_j^k)^t \, \underline{M}_j^k$ non-singular. Since equations 3.34 can always be solved for the eigenvalues, eq. 3.26a can always be inverse transformed such that the δ's are equal; and, consequently, a vector $\underline{\delta}$ can always be found which satisfies each of the constraints in eq. 3.25 simultaneously once a consistent set has been generated and solved for the vector $\underline{x}$.

Fortunately it is not necessary to solve for $\underline{\delta}$ explicitly. It is sufficient to realize that evaluating eq. 3.30 at the determined solution vector $\underline{x}$ yields numerical values:

$$
\begin{aligned}
\underline{x}^t \; {}_q\underline{D}_j^k \; \underline{x} &= k_k \\
\underline{x}^t \; {}_r\underline{D}_j^k \; \underline{x} &= k_k'
\end{aligned}
\qquad (3.35)
$$

If a particular scalar k_k is zero, then the corresponding $\underline{\delta}^t \underline{Q}_j^k \underline{\delta}$ $(\underline{\delta}^t \underline{R}_j^k \underline{\delta})$ is also zero; and if a particular k_k is positive, (the only other possible condition) then the corresponding $\underline{\delta}^t \underline{Q}_j^k \underline{\delta}$ $(\underline{\delta}^t \underline{R}_j^k \underline{\delta})$ are also positive although of different magnitude. This can be demonstrated as follows:

Let

$$\underline{\delta} = \underline{M}_j^k \; {}_m\underline{W}^k \; \underline{x}$$

which is

$$\underline{\delta} = l^k \underline{M}_j^k \underline{x} \tag{3.35a}$$

for

$$_{m}\underline{W}^k = l^k \underline{I}$$

Substituting eq. 3.35a in 3.24 yields:

$$\underline{x}^t (\underline{M}_j^k)^t (l^k)^2 \underline{Q}_j^k \underline{M}_j^k \underline{x} = l_0 (l^k)^2 x_0^{\ 2} + l_1 (l^k)^2 x_1^{\ 2} + \ldots + l_{j-1} (l^k)^2 x_{j-1}^{\ \ 2}$$

$$= (l^k)^2 [l_0 x_0^{\ 2} + l_1 x_1^{\ 2} + \ldots + l_{j-1} x_{j-1}^{\ \ 2}] \tag{3.35b}$$

Comparing eq. 3.35b with 3.28, it is evident that eq. 3.35b is eq. 3.28 multiplied by a positive constant which establishes the assertion above.

Recalling that the quadratic forms are actually the numerator coefficients of the real part of the augmented dp impedance, by solving eq. 3.31 and evaluating 3.30 the corresponding inequalities 3.24 are revealed as either zero or positive. It will now be demonstrated that this information is sufficient to determine the optimal augmenting and augmented polynomial to insure that the real part of $Z_a(s)$ is positive term by term.

C. OPTIMAL AUGMENTING AND AUGMENTED POLYNOMIALS

In this section knowledge of the form of the augmented driving-point impedance (i.e., which coefficients are positive and which zero) will lead to the development of the optimal augmenting polynomial and consequently the optimal augmented driving-point impedance in a straightforward way.

Consider an impedance function

$$Z(s) = \frac{m_1 + n_1}{m_2 + n_2} \tag{3.36}$$

and an augmenting polynomial

$$P(s) = m_1' + n_1'. \tag{3.36a}$$

Then

$$Z_a(s) = Z(s)\,\frac{P(s)}{P(s)} = \frac{m_1'' + n_1''}{m_2'' + n_2''} \tag{3.37}$$

is the augmented polynomial.

$$\mathrm{Re}Z_a(j\omega) = \left.\frac{m_1''m_2' - n_1''n_2''}{(m_2'')^2 - (n_2'')^2}\right|_{s=j\omega} \tag{3.38}$$

where

$$\begin{aligned} m_1'' &= m_1m_1' + n_1n_1' \\ m_2'' &= m_2m_1' + n_2n_1' \\ n_1'' &= m_1n_1' + n_1m_1' \\ n_2'' &= m_2n_1' + n_2m_1' \end{aligned} \tag{3.39}$$

Substituting eq. 3.39 in 3.38 yields

$$\mathrm{Re}Z_a(j\omega) = \left.\frac{(m_1' + n_1')\,(m_1' - n_1')\,(m_1m_2 - n_1n_2)}{(m_1' + n_1')\,(m_1' - n_1')\,(m_2^2 - n_2^2)}\right|_{s=j\omega} \tag{3.40}$$

$$\mathrm{Re}Z_a(j\omega) = \mathrm{Re}\left(\frac{P(j\omega)}{P(j\omega)}\right)\mathrm{Re}Z(j\omega) \tag{3.41}$$

$$\frac{\mathrm{Num}(\mathrm{Re}Z_a(j\omega))}{\mathrm{Den}(\mathrm{Re}Z_a(j\omega))} = \frac{\mathrm{Re}P(j\omega)\ \mathrm{Num}(\mathrm{Re}Z(j\omega))}{\mathrm{Den}(\mathrm{Re}Z_a(j\omega))} \tag{3.42}$$

where $\mathrm{Num}(\mathrm{Re}Z_a(j\omega))$ and $\mathrm{Den}(\mathrm{Re}Z_a(j\omega))$ represent numerator and denominator terms respectively. Therefore,

$$\mathrm{Num}(\mathrm{Re}Z_a(j\omega)) = \mathrm{Re}P(j\omega)\ \mathrm{Num}(\mathrm{Re}Z(j\omega)) \tag{3.43}$$

From eq. 3.24 we know which coefficients of $\mathrm{Num}(\mathrm{Re}Z_a(j\omega))$ are zero and which are positive, and we also know $\mathrm{Num}(\mathrm{Re}Z(j\omega))$, the original unaugmented dp impedance. A simple-minded but none the less effective

way to find $ReP(j\omega)$ and the unknown coefficients of $Num(ReZ_a(j\omega))$ is to proceed by long division. The scheme is best illustrated by a numerical example.

Given $Num(ReZ(j\omega)) = \omega^8 - 2\omega^6 + 3\omega^4 - 4\omega^2 + 5$, developing the form of $Num(ReZ_a(j\omega))$ using the Danzig method outlined above to solve eq. 3.31 for $\underline{z}$ and substituting into eq. 3.30, the following information is obtained concerning $Num(ReZ_a(j\omega))$

$$Num(ReZ_a(j\omega)) = \omega^{12}+0\omega^{10}+0\omega^{8}+0\omega^{6}+0\omega^{4}+c_5\omega^2+c_6,$$

where $c_5 > 0$ and $c_6 > 0$.

Dividing $Num(ReZ(j\omega))$ into $Num(ReZ_a(j\omega))$

$$\begin{array}{l|rrrrrrr}
 & 1 & +2 & +1 & & & & \\
\hline
1\ -2\ +3\ -4\ +5 & 1 & +0 & +0 & +0 & +0 & +c_5 & +c_6 \\
 & 1 & -2 & +3 & -4 & +5 & & \\
\cline{2-6}
 & & +2 & -3 & +4 & -5 & +c_5 & \\
 & & +2 & -4 & +6 & -8 & +10 & \\
\cline{3-7}
 & & & +1 & -2 & +3 & +(c_5-10) & +c_6 \\
 & & & +1 & -2 & +3 & -4 & +5 \\
\cline{4-8}
 & & & & & & (c_5-6) & (c_6-5)
\end{array}$$

and setting $c_5 = 6$ and $c_6 = 5$ the division terminates. The numerator of the $ReP(j\omega) = \omega^4 + 2\omega^2 + 1$ and the numerator of $ReZ_a(j\omega) = \omega^{12}+6\omega^2+5$. A feature of this method that can be useful is that the augmenting polynomial $ReP(j\omega)$ always has non-negative coefficients. It may even be Hurwitz. In the Miyata method this added constraint is not needed; but in transfer function synthesis, where augmentation may be called for, the augmenting polynomial is additionally required to be Hurwitz.

It turns out that if $Z(s)$ is of second order or less, it is unnecessary to have exact knowledge of the form $Num(ReZ_a(j\omega))$ since Poincare and Lewis[4] have demonstrated that for $Z(s)$ quadratic an augmented polynomial having but three non-zero positive terms can invariably be generated. For $Z(s)$ of greater than second order, in general, exact knowledge of the form of

$Num(ReZ_a(j\omega))$ is required and the preliminary steps which yield this form cannot be bypassed.

The problem is now essentially solved, and a brief outline of the computational steps is in order.

First generate a consistent set of non-linear inequalities (3.24) and then employ the transform 3.26a to set up and solve the resultant linear set of inequalities 3.31 by the Danzig simplex method which insures that as many as possible of the inequalities are zero, thereby minimizing the number of elements in the subsequent network synthesis. Armed with the solution vector $\underline{x}$, determine the form of the solved inequalities 3.35 that yields which coefficients of the numerator of the real part of the augmented dp impedance are zero and which are positive. Divide the given $Num(ReZ(j\omega))$ into $Num(ReZ_a(j\omega))$ and solve for the unknown positive coefficients. The process is now complete and the numerator of the real part is known (evaluating the denominator is straightforward), and one can proceed to the synthesis part of the problem with a polynomial that is positive term by term.

At this point it seems wise to demonstrate the solution algorithm and digital computer program developed to augment a moderately involved driving-point impedance.

IV. DIGITAL COMPUTER PROGRAM

From consideration of the computational scheme outlined, it is obvious that the technique is admirably suited to implementation on a digital computer. This is espacially true since solving a set of linear inequalities under the constraint that the solution variables be greater than zero is very similar to the basic linear programming problem which is the subject of the Mathematical Programming System[5] (MPS/360).

Therefore, a highly developed and efficient system is readily available to solve the linearized inequalities, and the programming problem becomes one of simply generating the inequalities from the original driving-point impedance and inputing the set into the MPS system.

The digital program required for systematic solution is straightforward and, aside from typical programming difficulties, simple. The entire solution process has not been consecutively programmed due to interface restrictions between OS/360 and MPS/360. The program follows in block diagram and Fortran format.

SUBROUTINE GROAN	Calculate the element values for the matrix of nonlinear inequalities.
n=1	
SUBROUTINE GRUNT	Form the matrix of inequalities.
SUBROUTINE GROWL	Convert the matrix of nonlinear inequalities to a matrix of linear inequalities.
SUBROUTINE PUNCH	Prepare an input data deck for the MPS/360 program which solves the linear inequalities and consequently the form of the original nonlinear inequalities.
n=n+1	
MPS/360	Output the required order of augmentation and form of the numerator of the real part of the driving-point impedance.

```
// EXEC FORTCLGP,REGION.GO=100K,TIME.GO=5
//FORT.SYSIN DD *
      DIMENSION ALFA(10),BETA(10),A(10),C(10),Q(10,10,10)
      DIMENSION R(10,10,10)
      DIMENSION QR(10,10),XX(10,36),YY(10,36)
    1 FORMAT(I10)
    2 FORMAT(8F10.2)
      READ (5,1) M
      READ (5,1) N
      M3= (M+2)/2
      M4=2*M3
      M5=4*M3
      MM=M+1
      N1=N+1
      N2=N1+M
      READ (5,2) (ALFA(I),I=1,MM)
      READ (5,2) (BETA(I),I=1,MM)
      CALL GROAN (M,N2,M3,M5,ALFA,BETA,A,C)
      DO3 N9=1,N1
      CALL GRUNT (M3,N9,A,C,Q,R)
      CALL GROWL (Q,R,M3,M4,N9,QR,XX,YY)
      CALL PUNCHY(N2,N9,M4,QR)
    3 CONTINUE
      STOP
      END


      SUBROUTINE GROAN (M,N2,M3,M5,ALFA,BETA,A,C)
      DIMENSION ALFA(10),BETA(10),A(10),C(10)
      MM=M+2
      DO 1 I=MM,M5
      ALFA(I)=0.0
    1 BETA(I)=0.0
      DO2 M1=1,M3
      A(M1)=0.0
      C(M1)=0.0
      M2=2*M1
      DO 3 N1=1,M2
      I=2*N1
      J=I-1
      K=4*M1-I
      L=K+1
      II=I-1
      JJ=J-1
      KK=K-1
      LL=L-1
      IF(K) 4,4,5
    4 K=MM
    5 IF(KK) 6,6,7
    6 KK=MM
    7 IF (LL) 8,8,9
    8 LL=MM
    9 IF (JJ) 10,10,11
   10 JJ=MM
   11 A(M1)=ALFA(I)*BETA(K)-ALFA(J)*BETA(L)+A(M1)
      C(M1)= ALFA(II)*BETA(KK)-ALFA(JJ)*BETA(LL)+C(M1)
    3 CONTINUE
    2 CONTINUE
      M6=M3+1
      DO12 I=M6,N2
      A(I)=0.0
   12 C(I)=0.0
      RETURN
      END


      SUBROUTINE GRUNT (M3,N9,A,C,Q,R)
      DIMENSION A(10),C(10),Q(10,10,10),R(10,10,10)
      DO 1 I=1,M3
      DO 2 J=1,N9
      DO 3 K=1,N9
      Q(I,J,K)=0.C
```

```
 3 R(I,J,K)=0.C
 2 CONTINUE
 1 CONTINUE
   DO 4 I=1,M3
   L=I
   N=2*I
   DO 5 J=1,N,2
   DO 6 K=J,N9,4
   Q(I,J,K)=A(L)
   Q(I,J+1,K+1)=C(L)
   Q(I,J,K+2)=-C(L)
   IF (J-1) 7,7,11
 7 Q(I,J+1,K+3)=-A(L)
   GO TO 12
11 Q(I,J+1,K+3)=-A(L-1)
12 IF(I.EQ.1) GO TO 4
   IF(L-1) 4,5,6
 6 L=L-1
 5 CONTINUE
 4 CONTINUE
   DO 15 I=1,M3
   L=I
   N=2*I
   DO 16 J=1,N,2
   DO 17 K=J,N9,4
   R(I,J,K)=C(L+1)
   R(I,J+1,K+1)=A(L)
   R(I,J+2,K+2)=C(L)
   R(I,J,K+2)=-A(L)
   R(I,J+1,K+3)=-C(L)
   IF(I.EQ.1) GO TO 15
   IF (L-1) 15,16,17
17 L=L-1
16 CONTINUE
15 CONTINUE
   DO 8 I=1,M3
   DO 9 J=1,N9
   M=J+1
   DO 10 K=M,N9
   Q(I,K,J)=Q(I,J,K)
10 R(I,K,J)=R(I,J,K)
 9 CONTINUE
 8 CONTINUE
   DO 20 I=1,M3
   DO 30 J=1,N9
30 WRITE(6,25)(Q(I,J,K),K=1,N9)
   DO 40 J=1,N9
40 WRITE(6,25)(R(I,J,K),K=1,N9)
20 CONTINUE
25 FORMAT(//,6F15.5)
   RETURN
   END

   SUBRCUTINE GROWL(Q,R,M3,M4,N9,QR,XX,YY)
   DIMENSION Q(10,10,10),R(10,10,10)
   DIMENSION QR(10,10),XX(10,40),YY(10,40)
   DIMENSION QC(10),RR(10),X(40),Y(40)
   DO 12 I=1,M3
   II=1
   DO 9 K=1,N9
   DO 10 J=1,K
   QQ(II)=Q(I,J,K)
   RR(II)=R(I,J,K)
10 II=II+1
 9 CONTINUE
   CALL EIGEX(CQ,X,N9,0)
   CALL EIGEX(RR,Y,N9,0)
   L=1
   III=2*I-1
   IIII=III+1
   DO 15 J=1,N9
```

```
      QR(III,J)=QQ(L)
      QR(IIII,J)=RR(L)
   15 L=J+L+1
      N10=N9**2
      DO 16 J=1,N10
      XX(I,J)=X(J)
   16 YY(I,J)=Y(J)
   12 CONTINUE
      DO 40 I=1,M4
   40 WRITE(6,30)(QR(I,J),J=1,N9)
      DO 45 I=1,M3
      WRITE(6,35)(XX(I,J),J=1,N10)
   45 WRITE(6,35)(YY(I,J),J=1,N10)
   30 FORMAT(//,6F15.5)
   35 FORMAT (//,6F15.5)
      RETURN
      END


      SUBROUTINE PUNCHY(N2,N9,M4,QR)
      DIMENSION QR(10,10),QR1(10,10)
   40 FORMAT('NAME',10X,'TESTPROB')
   41 FORMAT('ROWS')
   42 FORMAT (2X,'N COST')
   43 FORMAT(2X,'G ROW',I1)
   44 FORMAT('COLUMNS')
   45 FORMAT(4X,'COL1',6X,'COST',6X,'0.0')
   46 FORMAT(4X,'COL1',6X,'ROW',I1,6X,F12.5)
   47 FORMAT('RHS')
   48 FORMAT (4X,'RHS',7X,'COST',6X,'0.0')
   49 FORMAT(4X,'RHS',7X,'ROW',I1,6X,F12.5)
   50 FORMAT('ENDATA')
   60 FORMAT(4X,'COL',I1,6X,'COST',6X,'0.0')
   61 FORMAT(4X,'COL',I1,6X,'ROW',I1,6X,F12.5)
   62 FORMAT(4X,'RHS',7X,'ROW',I1,6X,F12.5)
      IF(N9.EQ.1) GO TO 70
      IF (N9.GT.2) GO TO 30
      WRITE(7,40)
      WRITE(7,41)
      WRITE(7,42)
      WRITE(7,43)(I,I=1,M4)
      WRITE(7,44)
      WRITE(7,45)
      WRITE(7,46)(I,QR(I,1),I=1,M4)
      WRITE(7,47)
      WRITE(7,48)
      DO 1 I=1,M4
    1 QR1(I,2)=-QR(I,2)
      WRITE(7,49)(I,QR1(I,2),I=1,M4)
      WRITE(7,50)
      GO TO 70
   30 WRITE(7,40)
      WRITE(7,41)
      WRITE(7,42)
      WRITE(7,43)(I,I=1,M4)
      WRITE(7,44)
      N8=N9-1
      WRITE(7,60)(I=1,N8)
      DO2 J=1,N8
    2 WRITE(7,61)(J,I,QR(I,J),I=1,M4)
      WRITE(7,47)
      WRITE(7,48)
      DO3 I=1,M4
    3 QR1(I,N9)=-QR(I,N9)
      WRITE(7,62)(I,QR1(I,N9),I=1,M4)
      WRITE(7,50)
   70 RETURN
      END
```

```
      SUBROUTINE EIGEX(A,R,N,MV)
      DIMENSION A(1),R(1)
    5 RANGE=1.0E-6
      IF(MV-1) 10,25,10
   10 IQ=-N
      DO 20 J=1,N
      IQ=IQ+N
      DO 20 I=1,N
      IJ=IQ+I
      R(IJ)=0.0
      IF(I-J) 20,15,20
   15 R(IJ)=1.0
   20 CONTINUE
   25 ANORM=0.0
      DO 35 I=1,N
      DO 35 J=I,N
      IF(I-J) 30,35,30
   30 IA=I+(J*J-J)/2
      ANORM=ANORM+A(IA)*A(IA)
   35 CONTINUE
      IF(ANORM) 165,165,40
   40 ANORM=1.414*SQRT(ANORM)
      ANRMX=ANORM*RANGE/FLOAT(N)
      IND=0
      THR=ANORM
   45 THR=THR/FLOAT(N)
   50 L=1
   55 M=L+1
   60 MQ=(M*M-M)/2
      LQ=(L*L-L)/2
      LM=L+MQ
   62 IF( ABS(A(LM))-THR) 130,65,65
   65 IND=1
      LL=L+LQ
      MM=M+MQ
      X=0.5*(A(LL)-A(MM))
   68 Y=-A(LM)/ SQRT(A(LM)*A(LM)+X*X)
      IF(X) 70,75,75
   70 Y=-Y
   75 SINX=Y/ SQRT(2.0*(1.0+( SQRT(1.0-Y*Y))))
      SINX2=SINX*SINX
   78 COSX= SQRT(1.0-SINX2)
      COSX2=COSX*COSX
      SINCS =SINX*COSX
      ILQ=N*(L-1)
      IMQ=N*(M-1)
      DO 125 I=1,N
      IQ=(I*I-I)/2
      IF(I-L) 80,115,80
   80 IF(I-M) 85,115,90
   85 IM=I+MQ
      GO TO 95
   90 IM=M+IQ
   95 IF(I-L) 100,105,105
  100 IL=I+LQ
      GO TO 110
  105 IL=L+IQ
  110 X=A(IL)*COSX-A(IM)*SINX
      A(IM)=A(IL)*SINX+A(IM)*COSX
      A(IL)=X
  115 IF(MV-1) 120,125,120
  120 ILR=ILQ+I
      IMR=IMQ+I
      X=R(ILR)*COSX-R(IMR)*SINX
      R(IMR)=R(ILR)*SINX+R(IMR)*COSX
      R(ILR)=X
  125 CONTINUE
      X=2.0*A(LM)*SINCS
```

```
      Y=A(LL)*COSX2+A(MM)*SINX2-X
      X=A(LL)*SINX2+A(MM)*COSX2+X
      A(LM)=(A(LL)-A(MM))*SINCS+A(LM)*(COSX2-SINX2)
      A(LL)=Y
      A(MM)=X
  130 IF(M-N) 135,140,135
  135 M=M+1
      GO TO 60
  140 IF(L-(N-1)) 145,150,145
  145 L=L+1
      GO TO 55
  150 IF(IND-1) 160,155,160
  155 IND=0
      GO TO 50
  160 IF(THR-ANRMX) 165,165,45
  165 RETURN
      END
//GO.SYSIN DD *
```

```
//JOBLIB    DD  DSNAME=SYS1.MPS360LP,DISP=(SHR,PASS)
//              EXEC LINPROG
//MPS1.SYSIN DD *
               PROGRAM
               INITIALZ
               MOVE(XPBNAME,'PBFILE')
               MOVE(XDATA,'TESTPROB')
               MOVE(XOBJ,'COST')
               MOVE(XRHS,'RHS')
               MVADR(XDONFS,INFEAS)
               MVADR(XDOFEAS,FEAS)
               GOTO(AARRRG)
INFEAS         MOVE(XPBNAME,'PBFILE')
               MOVE(XDATA,'TESTPROB')
               CONVERT('SUMMARY')
               GOTO(AAARRG)
AARRRG         CONVERT('SUMMARY')
AAARRG         BCDOUT
               SETUP('MAX')
               PRIMAL
FEAS           SOLUTION
               EXIT
               PEND
//MPS2.SYSIN DD *
```

V. SAMPLE PROBLEM AND SOLUTION

$$Z(s) = \frac{s^2 + 1/2s + 1}{s^2 + \quad s + 1}$$

From computer program results

$$NRez_a(j\omega) = \omega^{10} + 0\omega^8 + 0\omega^6 + 0\omega^4 + c_4\omega^2 + c_5$$

$$NRez(j\omega) = \omega^4 - 3/2\omega^2 + 1$$

$$DReZ(j\omega) = \omega^4 - \omega^2 + 1$$

Dividing $NReZ_a(j\omega)$ by $NReZ(j\omega)$ gives:

$$
\begin{array}{l|rrrrrr}
 & 1 & +3/2 & +5/4 & +3/8 & & \\
\hline
1\ -3/2 + 1 & 1 & 0 & 0 & 0 & +c_4 & +c_5 \\
 & 1 & -3/2 & 1 & & & \\
\cline{2-4}
 & & 3/2 & -1 & & & \\
 & & 3/2 & -9/4 & +3/2 & & \\
\cline{3-5}
 & & & +5/4 & -3/2 & & \\
 & & & +5/4 & -15/8 & +5/4 & \\
\cline{4-6}
 & & & & +3/8 & -5/4 & \\
 & & & & +3/8 & -9/16 & +3/8 \\
\cline{5-7}
 & & & & & -11/16 & -3/8
\end{array}
$$

Choosing c_4 and c_5 equal to 11/16 and 3/8 respectively, the division terminates and the real-part numerator of the augmenting polynomial is seen to be

$$\omega^6 + 3/2\omega^4 + 5/4\omega^2 + 3/8.$$

The real-part numerator of the augmented polynomial has three non-zero terms and reads

$$\omega^{10} + 11/16\omega^2 + 3/8.$$

The denominator is

$$DReZ(j\omega) \times ReP(j\omega)$$

which equals

$$\omega^{10} + 1/2\omega^8 + 3/4\omega^6 + 5/8\omega^4 + 7/8\omega^2 + 3/8.$$

Therefore

$$\operatorname{Re} Z_a(j\omega) = \frac{\omega^{10}+11/16\omega^2+3/8}{\omega^{10}+1/2\omega^8+3/4\omega^6+5/8\omega^4+7/8\omega^2+3/8}$$

which is positive for all ω, as required.

TABLE I[6]

COMPARISON OF THE NUMBER OF CIRCUIT ELEMENTS REQUIRED FOR SINGLE-N AND 1/2-N SPLIT MIYATA AND MODIFIED BOTT-DUFFIN METHODS

	Single-n Split		Half-n Split		Modified Bott-Duffin	
n	n_{lc}	n_r	n_{lc}	n_r	n_{lc}	n_r
1	1	2	1	2	1	2
2	4	3	4	3	5	3
3	9	4	6	4	7	5
4	16	5	10	5	15	7
5	25	6	14	6	19	11
6	36	7	17	7	35	15
8	64	9	25	9	75	31
10	100	11	35	11	155	63
12	144	13	44	13	315	127
14	196	15	52	15	635	255
16	256	17	62	17	1275	511
18	324	19	74	19	2555	1023
20	400	21	86	21	5115	2047
30	900	31	139	31	163,835	65,535

n_{lc} = number of reactive components

n_r = number of resistive components

<u>TABLE II</u>

GENERAL POLYNOMIAL EXPANSIONS OF FIRST THROUGH FIFTH ORDER AUGMENTATIONS

$k = 0,1,\ldots,(N-1)/2$ for a_k

$k = 0,1,\ldots,N/2$ for c_k

For non-integer values round down to the next lower integer.

$$\underline{\delta_1 s+\delta_0}$$

$$\left.\begin{array}{l} a_k\delta_0^2+c_k\delta_1^2 \\ c_{k+1}\delta_0^2+a_k\delta_1^2 \end{array}\right\} \geqq 0$$

$$\underline{\delta_2 s^2+\delta_1 s+\delta_0}$$

$$\left.\begin{array}{l} a_k\delta_0^2+c_k\delta_1^2+a_{k-1}\delta_2^2-2c_k\delta_0\delta_2 \\ c_{k+1}\delta_0^2+a_k\delta_1^2+c_k\delta_2^2-2a_k\delta_0\delta_2 \end{array}\right\} \geqq 0$$

$$\underline{\delta_3 s^3+\delta_2 s^2+\delta_1 s+\delta_0}$$

$$\left.\begin{array}{l} a_k\delta_0^2+c_k\delta_1^2+a_{k-1}\delta_2^2+c_{k-1}\delta_3^2-2c_k\delta_0\delta_2-2a_k\delta_1\delta_3 \\ c_{k+1}\delta_0^2+a_k\delta_1^2+c_k\delta_2^2+a_{k-1}\delta_3^2-2a_k\delta_0\delta_2-2c_k\delta_1\delta_3 \end{array}\right\} \geqq 0$$

$$\underline{\delta_4 s^4+\delta_3 s^3+\delta_2 s^2+\delta_1 s+\delta_0}$$

$$\left.\begin{array}{l} a_k\delta_0^2+c_k\delta_1^2+a_{k-1}\delta_2^2+c_{k-1}\delta_3^2+a_{k-2}\delta_4^2-2c_k\delta_0\delta_2-a_k\delta_1\delta_3+2a_{k-1}\delta_0\delta_4-2c_{k-1}\delta_2\delta_4 \\ c_{k+1}\delta_0^2+a_k\delta_1^2+c_k\delta_2^2+a_{k-1}\delta_3^2+c_{k-1}\delta_4^2-2a_k\delta_0\delta_2-2c_k\delta_1\delta_3+2c_k\delta_0\delta_4-2a_{k-1}\delta_2\delta_4 \end{array}\right\} \geqq 0$$

$$\underline{\delta_5 s^5+\delta_4 s^4+\delta_3 s^3+\delta_2 s^2+\delta_1 s+\delta_0}$$

$$\left.\begin{array}{l} a_k\delta_0^2+c_k\delta_1^2+a_{k-1}\delta_2^2+c_{k-1}\delta_3^2+a_{k-2}\delta_4^2+c_{k-2}\delta_5^2-2c_k\delta_0\delta_2-2a_k\delta_1\delta_3+2a_{k-1}\delta_0\delta_4-2c_{k-1}\delta_2\delta_4+2c_{k-1}\delta_1\delta_5-2a_{k-2}\delta_3\delta_5 \\ c_{k+1}\delta_0^2+a_k\delta_1^2+c_k\delta_2^2+a_{k-1}\delta_3^2+c_{k-1}\delta_4^2+a_{k-2}\delta_4^2-2a_k\delta_0\delta_2-2c_k\delta_1\delta_3+2c_k\delta_0\delta_4-2a_{k-1}\delta_2\delta_4+2a_{k-1}\delta_1\delta_5-2c_{k-1}\delta_3\delta_5 \end{array}\right\} \geqq 0$$

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13. ABSTRACT

These investigations generalize Miyata's synthesis of passive driving-point impedance functions by developing a step-by-step computational technique for augmenting a general driving-point impedance to insure that the real part of the augmented impedance is positive term by term. This goal has been accomplished and programmed under the provision that the real part of the original impedance is minimum reactive, i.e., has no zeros on the jω-axis. The latter requirement is not restrictive since zeros on the jω-axis can be removed a priori.

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